An Accuracy Evaluation of Multibody Dynamics for the Knee Support Exoskeleton Model with Respect to Implicit Methods for Numerical Integration

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Abstract

Numerical integration takes an important role to analyze models described by ordinary differential equations and it largely contributes to an assurance of the accuracy in displacement analyses of multibody dynamics if the model consists of several bodies especially with dynamic components such as springs and dampers. Exoskeletal assistive devices require flexible materials for absorbing reaction forces from human joints, which implies an inevitable necessity of an accurate evaluation of elastic effects in the model. In the present study, we introduced implicit methods for numerical integration to analyze the model under the formulation of multibody dynamics and computer experiments demonstrated results from explicit and implicit methods for the numerical integration as a comparison analysis. It can improve a degree of accuracy in inverse dynamics even in the theory of flexible Multibody Dynamics (fMBD).

*Keywords*: Numerical integration, Multibody dynamics (MBD), Analysis error, Ordinary differential equation

# Introduction

For the sake of analyses of kinematics and kinetics that consist of multiple bodies known as the multibody systems, multibody dynamics (MBD) analysis is getting to be the standard method [[1]](#R1), [[2]](#R2), [[3]](#R3), [[4]](#R4), [[5]](#R5). Thus, the description of ordinary differential equations accompanied with an appropriate numerical integration acts an important role in ensuring accuracy, especially in the displacement analysis of systems, which are incorporating dynamic elements such as spring and damper components [[6]](#R6). Indeed, flexible materials are of interest for consideration of bodies absorbing and utilizing reaction force. In consideration of human joint models, those factors are inevitable not only for the detailed analysis of the joint mechanisms but also development of exoskeleton type support device which assists human movement. Therefore, it is highly important to reconstruct an accurate computer experiment even with elastic effects in the model. An increase in the number of bodies and elastic components provides a high complexity in MBD analysis, which may cause the numerical error in the computer experiment.

In particular to dynamics analysis with moving bodies, the selection of the numerical integration method is crucial for the realization of the actual dynamics occurring in the real world. It cannot be solved by a simple way to chip the time step of the integration in the explicit numerical method. In the implicit numerical integration, it will help to refine the time step adaptively.

In the present study, we introduced the implicit numerical integration methods under the MBD formulation [[7]](#R7) and evaluate the effect in comparative analysis between results of explicit and implicit solutions of numerical integrations [[7]](#R7), [[8]](#R8), [[9]](#R9). In this sense, the advantages to utilize the implicit methods can be revealed in the computer experiment, which realizes the necessity and appropriate selection of the numerical method depending on the target system. It will contribute to an actual numerical solution not only for rigid-body mechanics but also for flexible multibody dynamics (fMBD), which embeds the finite element method (FEM) into the original description of MBD.

# Methodology

## Numerical error under MBD analysis

In MBD-based dynamic analysis, the coordinates and rotation angles of each mechanical element in the analytical system are obtained by solving the second-order ordinary differential equations based on the following differential algebraic equations for the generalized acceleration matrix . The meanings of the characters in Eq. (1) are as [Table 1](#table1).

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|  | (1) |

Table 1 The planning and control components.

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|  | Mass matrix |
|  | Jacobian matrix differentiated from constraint equation in generalized coordinates |
|  | Generalized acceleration matrix |
|  | Lagrange multiplier |
|  | Generalized force |
|  | Acceleration equation |

In computer analysis, this second-order ordinary differential equation can be solved by numerical integration methods, however errors accumulate at each analysis step. These numerical errors appear in the deviations of nodal coordinates connecting each element in the kinematic analysis, and the accuracy of the numerical integration methods can be evaluated by the distance of the nodal coordinates ([Figure 1](#Fig1)).

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| Figure 1 Displacements of the nodal coordinates caused by errors of numerical integration in knee linkage model [[6]](#R6). |

## Numerical integration methods

In order to compare and verify the analytical errors in the MBD analysis, we applied the numerical integration methods to the dynamic analysis with MBD of single and double pendulums ([Figure 2](#Fig2)) to simply verify the errors.

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| 黒い背景に白い文字がある  中程度の精度で自動的に生成された説明 | 夜に光っている月  AI によって生成されたコンテンツは間違っている可能性があります。 |
| 1. Single pendulum. | 1. Double pendulum. |
| Figure 2 Generalized coordinate systems of pendulums for MBD analysis. | |

For comparing errors caused by numerical integration methods, we applied Runge-Kutta Gill’s method [[10]](#R10), the two-stage fourth order and the three-stage sixth order implicit Runge-Kutta (IRK) method. Gill’s method is explicit fourth order Runge-Kutta method. The s-stage IRK method is described as the following equation [[7]](#R7), [[8]](#R8). The coefficients *a, b* and *c* in Eq. (2) are due to the Butcher array ([Table 2](#table2)).

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| \documentclass{article} \usepackage{amsmath} \pagestyle{empty} \begin{document}  $ \left\{ \begin{array}{l} k_i=f(x_0 + c_i h, y_0+h \sum\limits_{j=1}^{s} a_{ij} k_j) \\ y_{(x_0+h)}=y_0+h \sum\limits_{i=1}^{s} b_{i} k_i \\ \end{array} \right. $   \end{document} | (2) |

Table 2 The Butcher array for IRK.

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In MBD analysis, since the generalized acceleration matrix is obtained by solving a second-order ordinary differential equation as Eq. (3), the equations for obtaining the generalized coordinates matrix and the generalized velocity matrix with IRK are as Eq. (4) and Eq. (5).

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|  | (3) |
| \documentclass{article} \usepackage{amsmath} \pagestyle{empty} \begin{document}  $ \left\{ \begin{array}{l} k_i=f(t_n + c_i h, q_n+h \sum\limits_{j=1}^{s} a_{ij} l_j, \dot{q}_n+h \sum\limits_{j=1}^{s} a_{ij} k_j) \\ l_i=\dot{q}_n+h \sum\limits_{j=1}^{s} a_{ij} k_j \end{array} \right. $   \end{document} | (4) |
| \documentclass{article} \usepackage{amsmath} \pagestyle{empty} \begin{document}  $ \left\{ \begin{array}{l} y'_{n+1}=p_{n+1}=P_n+h \sum\limits_{i=1}^{s} b_{i} k_i \\ y_{n+1}=y_n+h \sum\limits_{i=1}^{s} b_{i} l_i \end{array} \right. $   \end{document} | (5) |

Within each integral computation step, the Newton-Raphson method is inserted to find solutions of the simultaneous equations to derive all coefficients and in Eq. (4), then *q* and are obtained by Eq. (5). The Butcher arrays for 2stage 4order IRK ([Table 3](#table3)) and 3stage 6order IRK ([Table 4](#table4)) are shown as follows [[8]](#R8).

Table 3 The Butcher array for 2-4 IRK.

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Table 4 The Butcher array for 3-6 IRK.

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Each numerical integration method was implemented in the numerical computation of MBD analysis of single and double pendulums (time increment ), and the calculation errors were verified. The differences in the trajectory by numerical integrations are shown in [Figure 3](#Fig3).

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| 1. Single pendulum. |
| ダイアグラム  自動的に生成された説明 |
| 1. Double pendulum.   Figure 3 Trajectories of pendulums. |

[Figure 4](#Fig4) shows the step-by-step transition of the joint coordinate error computed by numerical integrations.

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| 1. Single pendulum. |
| グラフ  自動的に生成された説明 |
| 1. Double pendulum. |
| Figure 4 Comparison of coordinate errors by numerical integration method. |

According to these figures, the errors in the single and double pendulum are smaller for the implicit solution than for the explicit solution. It is also clear that the analytical error becomes smaller as the order of the numerical integration method increases. Therefore, in the analysis of a system consisting of multiple elements based on MBD, it is possible to evaluate the accuracy of numerical integral calculations from the errors in the joint coordinates of the mechanical elements constituting the system to be analyzed.

## Performance of the numerical integration on MBD analysis

In numerical computation, the analytical time is important as analytical accuracy. [Figure 5](#Fig5) shows the changes in numerical computation time due to the expansion of the order of the implicit numerical integration and the number of variables in the MBD dynamics analysis of the pendulums. The number of computation steps is 1500.

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| Figure 5 The analysis times of numerical integrations. |

The analysis times increase significantly when the number of steps in the implicit integral is expanded. In the implicit method, the solutions of simultaneous equations are obtained by inserted Newton-Raphson method, and numerical substitutions into symbolic matrix within this process require the computation processing time. The symbolic matrix is enlarged by increasing the number of implicit stages, which takes more computation time. Therefore, it is important to implement an efficient substitution process into a symbolic matrix for the implementation of higher order implicit methods in MBD analysis.

# Conclusion

The validation of the numerical integration in the pendulum model analysis with MBD shows methods to suppress the error due to the numerical integration of the second-order ordinary differential equations in the kinetic and dynamic analysis with MBD analysis.

In further analysis, these numerical integration methods can be adopted to the dynamics/inverse dynamics of knee joint mechanism and human walking analysis based on fMBD analysis, and comparisons of their analytical accuracy can be discussed. However, there are issues in implementing the higher order IRK to multielement mechanism analysis, such as improving the performance by streamlining the substitution process for symbolic matrix formulas in MATLAB.

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